Experimental and Analytical Investigation of Dispersed Flow Heat Transfer

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Experiments were carried out to understand and to quantify the enhancement of heat transfer in dispersed flows. Results showed that in the presence of glass particles 30 µm and 100 µm in diameter entrained in air, heat transfer coefficients were significantly increased. Relative enhancement was accentuated when the mass flow rate ratio of solids to air was increased, but it diminished when flow Reynolds number was increased. Also, smaller particles were found to be more effective in augmenting the heat transfer. An enhancement of as much as 170% was observed with 30 µm particles, whereas maximum enhancement with larger particles was 25%. The enhancement in heat transfer with large particles ($D_p > 100 \mu m$) is attributed to an increase in the turbulent intensity of the flow. A mechanistic model has been developed to describe the transport processes in a turbulent dispersed flow of large particles. Expressions have been derived for eddy diffusivities of momentum and heat. The model predictions compare quite favorably with the bubbly flow data reported in the literature as well as the data obtained in the present study. The results of this work should be useful in many engineering applications such as entrained flow of droplets downstream of a quench front formed during reflooding of an overheated nuclear reactor core.

Keywords: two-phase flow, dispersed flow, heat transfer enhancement

INTRODUCTION

There are numerous industrial applications in which dispersed flows are encountered. In dispersed flows, particles of a second or discontinuous phase are entrained in the primary or continuous phase. Examples of such flows are gas bubbles carried in a liquid, droplets entrained in a gas, and solid particles suspended in a gas or liquid. The present study has been motivated by the need to develop an understanding of the mechanisms responsible for enhancement of heat transfer in dispersed flows in tubes or rod bundles. Such a flow can exist downstream of the quench front during flooding of the reactor core after a hypothetical loss-of-coolant accident in a nuclear reactor.

In dispersed flows, many parameters such as density, size, and thermal capacity of the particles influence the heat transport process. Since an essential objective of this study is to investigate the effects of particle size in the dispersed flow, it would be convenient to distinguish two size regimes, hereafter referred to as fine particle and coarse particle sizes. Although the domain of each regime can be influenced by the flow conditions, here it is assumed that the diameter of the fine particles is generally less than 100 µm, and the coarse particles range in size from 100 µm to 2000 µm.

The effects of fine particles on the transport characteristics of dispersed flow have been widely discussed by various authors. An extensive literature survey has been presented by Depew and Kramer [1]. The earliest quantitative study on particulate flows is that of Farbar and Morley [2], who entrained alumina-silica catalyst particles in air flowing through a vertical tube and evaluated the overall heat transfer coefficient. It was observed that by increasing the mass flow rate of the solids the heat transfer coefficient could be increased. However, the level of enhancement was found to decrease with the flow Reynolds number. For this mixture of particles in the size range of 10-200 µm, Farbar and Morley correlated their data as

$$N_u = 0.14 \text{Re}^{0.9} (W_s/W_a)^{0.45}$$  \hspace{1cm} (1)

The wide range of particle sizes used in their experiments limited any conclusion about the influence of particle size on heat transfer enhancement.

Subsequently, Farbar and Depew [3] carried out experiments with nearly uniform size particles of 30, 70, 140, and 200 µm diameter. In these experiments, they observed that the smallest particles resulted in enhancements as high as 230% at a solid-to-air mass flow rate ratio of 6, whereas the largest particles had no appreciable effect. Another comparable study by Wilkinson and Norman [4] with glass particles of 40, 60, and 120 µm substantiated the observation of Farbar and Depew.

Also, general conclusions that have been drawn from the
results of various investigators are that the thermal entry length increases in the presence of solid particles and that this increase is more pronounced with smaller particles.

Despite the qualitative similarity of the results of various investigators, there is not always quantitative agreement. As mentioned earlier, various physical parameters such as particle size, tube diameter, and velocity of the continuous phase can influence the nature of the dispersed flow. Furthermore, lack of consistency in choice of an appropriate heat transfer coefficient has limited the use of some of the available data. As an example, some authors have given correlations based on overall heat transfer coefficients, and others have chosen arbitrary datum points downstream of the test section inlet for evaluating the heat transfer coefficients. The inadequacy of the available correlations in explaining the wide range of reported data clearly indicates that a more fundamental study on the turbulence structure of the flow is necessary.

It has been generally agreed that for fine particles the migration of the solid particles toward the wall tends to reduce the effective thickness of the viscous sublayer and decrease the thermal resistance of the flow. Briller and Peskin [5] proposed that the extent of disturbance of the viscous sublayer depends on three parameters: $\delta_v$, the dimensionless viscous sublayer thickness; $n_p$, the number density of the particles, which represents the effectiveness of the presence of the discontinuous phase in the continuous phase; and $p_d$, the penetration depth, which represents the level of the disturbance that can be expected from the particles. By incorporating these quantities into a nondimensional group, namely $n_p^{2/3} \delta_v \delta_d$, Briller and Peskin [5] proceeded to correlate a wide range of available data on solid-gas flows. The expression for the enhancement in heat transfer coefficient was obtained

$$\frac{N_u}{N_{u0}} = 0.0195 (n_p^{2/3} \delta_v \delta_d)^{0.6}$$

Equation (2) was only partially successful in predicting the highly scattered data. However, Depew and Kramer [1] showed that this expression gives a dependence of $(W_r/W_a)^{0.5}$ with the loading ratio, which is about the same as the power dependency given by the earlier correlation of Farbar and Morley [2].

In an exploratory investigation, Lee and Durst [6] studied the dynamics of the dispersed phase in a turbulent solid-air flow. They used a Laser Doppler Anemometry (LDA) technique for measuring the turbulence intensity in a vertical pipe. Their experiments were carried out with 100, 200, 400, and 800 $\mu$m glass particles entrained in air. Results indicated that for large particle sizes a particle-free region clearly existed near the wall and the dispersed phase tended to flow in the central portion of the pipe. This observation can explain the absence of appreciable enhancement of the heat transfer coefficient that has been reported for large particles.

As discussed earlier, dispersed flows of coarse size consist of particles that are about 100 $\mu$m in diameter or larger. This includes gas–liquid flows and those solid–gas flows in which the particle diameter falls within this range. A good number of studies documenting the two-phase gas–liquid heat transfer coefficient have been reported in the literature. These studies, which have been reviewed by Drucker et al [7], among others, unequivocally showed that the presence of a small amount of gas in a flowing liquid can substantially enhance the heat transfer from the wall.

Theofanous and Sullivan [8] conducted a series of experiments on turbulence intensity in a bubbly flow. They reported that an increase in the volume fraction of the entrained particles tended to increase the turbulence intensity in the flow. This observation is similar to the findings of Lee and Durst [6], which indicated an enhancement of turbulence intensity in dispersed flow. Theofanous and Sullivan [8] argued that the disturbance due to the presence of the particles is caused by their relative velocity with respect to the fluid. The shear work of this relative velocity acts as a source of turbulence energy production in the flow.

Based on this premise, they theoretically found a relationship between turbulence intensity and the macroscopic quantities of the flow as

$$\left( \frac{U_2}{U_c} \right)^2 = \frac{C}{2} (1 + \beta)(1 - \beta) + \lambda \beta (1 - \beta) \left( 1 - \frac{\rho_d}{\rho_c} \right) \frac{R_g}{2U_c^2}$$

In Eq. (3), the plus and minus signs are for upflow and downflow, respectively. The overbar represents time average.

Drucker et al [7], on the other hand, assumed that the wall shear stress under two-phase flow conditions was the sum of the shear stress for single-phase flow and the shear stress arising from the interaction between the two phases. By invoking Reynolds’s analogy, they found that the ratio of two-phase to single-phase heat transfer coefficients could be written as

$$\psi = 1 + f(\beta, Gr, Re)$$

The function $f$ was obtained by correlating a large variety of available bubbly flow heat transfer data in tubes and their own data in rod bundles as

$$\psi = 1 + C \left( \frac{\beta}{Gr} \frac{Re}{Re_c} \right)^{0.5}$$

The constant $C$ was found to have a value of 2.5 for tubes and was about 30% higher in rod bundles.

The studies of particulate flows and bubbly flows have been conducted exclusive of each other, and no attempt has been made in the past to find the unifying features of the two. Thus the broader objectives of the present work were:

1. To determine the enhancement in heat transfer for particulate flows in rod bundles. A detailed description of the experimental results is given in Ref. 9. The particulate flow represents a better simulation of droplet flow encountered downstream of the quench front during flooding.

2. To develop a model for the enhancement of heat transfer in dispersed flows. The focus here is on larger particles ($D_p \geq 100 \mu$m) so that similarities between bubbly and particulate flows can be highlighted.

**EXPERIMENTAL APPARATUS AND PROCEDURE**

**Experimental Apparatus**

The experiments with solids entrained in gas were carried out in the apparatus shown in Fig. 1. The main components of this apparatus are the test section, mixing chamber, variable area nozzle, and filtering system.

The test section consists of four rods arranged in a square
grid and placed parallel to each other inside a 5.08 cm I.D. plexiglass tube. The rods are 1.1 cm O.D. stainless steel tubes with a length of 184.2 cm and a wall thickness of 0.84 mm. A cross-sectional view of this assembly is presented in Fig. 2. Although Fig. 2 shows blockages around the rods, the present data were obtained in the absence of blockages.

Two of the four rods are instrumented with 30 gauge chromel-alumel thermocouples. Twenty thermocouples are spot welded on the inner wall of the stainless steel tubes, and the simulated rods are positioned such that thermocouples on one of the instrumented tubes face the inner channel while those on the other rod face the outer channel.

The thermocouples on each of the instrumented tubes are separately connected to the input terminals of a Fluke 2280-A programmable data logger. This device, which is equipped with an internal zero junction compensator, scans each array of thermocouples in 12 s and records the data on a printer. Data acquisition can be performed intermittently at prespecified intervals for any duration of time.

**Experimental Procedure**

Each experiment was carried out in two stages: measurement of single-phase flow heat transfer coefficients and measurement of solid-air flow heat transfer coefficients.

A variac in conjunction with a digital multimeter was used to

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**Figure 1.** Schematic presentation of experimental apparatus.

**Figure 2.** Cross-sectional view of four-rod bundle in the test section.
adjust the current drawn from the transformer to a value of 60 or 90 A. The choice of these values was based on obtaining the most stable current from the transformer and a large enough temperature difference between the wall and the fluid to minimize the relative magnitude of uncertainty in measured temperatures. Subsequently, thermocouple readings at an arbitrary location on the wall were monitored with the data logger. After the observed value had remained fairly constant for 1 min, the scanning and recording mode was started. The two arrays of thermocouples in the inner and outer channels were scanned every 30 s, and three sets of data were recorded for each channel. It is worth mentioning that by intermittent manual adjustment of the rotameters to a constant flow rate, steady flow of air was maintained throughout the experiment.

After the single-phase flow data were recorded, the solid–air flow was initiated. The control valve connected to the feeder tank was opened, and glass particles were gravity fed into the mixing chamber. An appropriate nozzle was installed downstream of the valve to set the solid mass flow rate at the desired rating. After the solid flow rate established itself, a chronometer was started, and the instantaneous weight of the tank was recorded. Subsequently, after ever 5 or 10 lb reduction in the weight of the feeder tank, the total weight and the corresponding time were recorded. Meanwhile the wall temperature was monitored continuously to ensure steady-state operation. As a result of some oscillations in the flow rate, only a pseudosteady state around a mean value associated with fluctuations in the local wall temperature was obtainable. Depending on the duration of the solid flow, a series of 6–12 sets of data were acquired for each array of thermocouples. Also during the course of data acquisition, electric current through the bundle was monitored by a multimeter and recorded manually.

Data Reduction

The acquired data were the axial temperature of the wall in the inner and outer channels and the air inlet temperature. For each channel, between 3 and 12 sets of data were recorded. After the data were gathered, the value of the wall temperature (in microvolts) associated with each axial location was found by averaging the samples. The solid flow rate was calculated by evaluating the time interval associated with the weight reduction of the feeder tank. For each experiment, between 5 and 14 samples of such data were reduced. After evaluation of the flow rate for each sample, the mean value and the standard deviation were determined. It was found that deviations were within 3–8% of the mean values.

Evaluation of the heat transfer coefficient required the bulk temperature at locations of thermocouples. Knowing the electric current \( I \), ohmic resistance of the rods \( R_0 \), inlet temperature of the mixture \( T_{in} \), length of the rod \( L \), and mass flow rates of solids and air, the bulk mixture temperature at any axial location was calculated as

\[
T_{b}(z) = T_{in} + \frac{I^2 R_0}{W_e C_{pa} + W_s C_{pm}} \left( \frac{z}{L} \right)
\]

Ohmic resistance of the rods was measured with a Wheatstone bridge before assembly of the bundle.

In writing Eq. (6), it is assumed that the fluid and particles are at the same temperature. Based on heat transfer analysis on a single particle submerged in a fluid with varying temperature in the direction of the flow, it was found that for the particle sizes used in the present experiments this assumption results in a maximum error of 6.5% in evaluating the heat transfer coefficient.

Once the bulk temperature was known, the heat transfer coefficient could be readily calculated as

\[
h = \frac{1}{4} \frac{I^2 R_0}{(T_{in}(z) - T_b(z)) A_{rod}}
\]

Both systematic and random errors contributed to the overall uncertainty in the heat transfer coefficient. The systematic errors were associated with the thermocouple readings, the weighing scale, the flowmeters, and the multimeter. The random errors resulted from the unsteadiness of the air and solid flow rates, which in turn caused fluctuations in the wall and bulk temperatures. For the random errors, the data were recorded over a period of time, and mean and standard deviations of the data were calculated. The standard deviation of the data excluding the systematic errors was used as a measure of the random error. The random errors contributed the most to the uncertainty in the present heat transfer data.

The maximum possible total error in the heat transfer coefficient is calculated to be ±23% for the two-phase flow and ±11.5% for the single-phase flow. In most of the reported experiments, however, the uncertainty was much less than the calculated maximum value.

RESULTS AND DISCUSSION

Experiments on heat transfer in dispersed flow of glass particles with mean diameter of 30 \( \mu \)m and 100 \( \mu \)m were carried out at Reynolds numbers from 14,000 to 21,000 with solid volume fractions varying from 0.0 to 0.003. The diameter of the small particles varied between 10 and 57 \( \mu \)m and that of the large particles between 88 and 100 \( \mu \)m, with the majority of the large particles lying in the 100 \( \mu \)m range. The ranges of Reynolds number and solid fraction are representative of the capabilities of the experimental apparatus. In Fig. 3, axial variation of Nusselt number of dispersed flow of 100 \( \mu \)m particles is shown as a function of the number of hydraulic diameters from the inlet. The heat transfer coefficient is found to increase with the increase in mass flow rate of the solids.

In Figs. 4 and 5, axial variations of the relative enhancement in heat transfer are shown for 30 and 100 \( \mu \)m particles, respectively. The plotted data are for the upper half of the rod bundle. It is seen that the maximum relative enhancement for the 100 \( \mu \)m particles is 25% as compared to 170% for the 30 \( \mu \)m particles. A comparison of the data for different Reynolds numbers showed that enhancement level decreased as the Reynolds number of the flow was increased.

Correlation of the Data

Briller and Peskin [5] had argued that in a dispersed flow the particles tend to deposit on the wall and enhance the heat transfer by disturbance of the viscous sublayer. As a result, the enhancement in heat transfer was correlated with a product of particle number density, viscous sublayer thickness, and penetration depth of the particles. The product is related to the macroscopic parameters of the flow as

\[
n_p^{2/3} \rho_{pd} = \frac{38.7(W_e/W_s)^{2/3}(D_e/D_p)^{1/3}(D_e/D_p)^{0.3}}{Re^{0.175}}
\]
enhancement for smaller particles is reasonably well represented with the parameter of Briller and Peskin. The best fit to the data is given by

$$\psi = 0.00863 (n_p^{2/3} \delta_h \rho d)^{0.75}$$

(9)

Combining Eqs. (8) and (9), it is found that the enhancement increases as $(W_s/W_o)^{0.5}$. This dependency is similar to that reported by Farbar and Morley [2].

On the other hand, the data for 100 μm particles do not appear to correlate with the parameter of Briller and Peskin. As noted earlier, the enhancement in heat transfer with 100 μm particles is much smaller than with 30 μm particles.

In Fig. 7, the enhancement data are plotted against the parameter $\beta \, Gr/Re^2$ suggested by Drucker et al [7]. The data for 100 μm diameter particles appear to follow the trend given by Eq. (5), which correlated a large variety of bubbly flow data. The magnitude of enhancement, however, is slightly lower. The data for 30 μm particles show a much stronger dependence on $\beta \, Gr/Re^2$ and appear not to correlate with the equation of Drucker et al [7] [Eq. (5)].

Thus there is strong evidence that the enhancement for the small particles ($D_p < 100 \, \mu m$) is due to their influence on the viscous sublayer. The larger particles, on the other hand, influence the heat transfer coefficient by a mechanism similar to the bubbly flows. Subsequently, an attempt is made here to model the enhancement of the heat transfer coefficient for large particles.

**A Model for Heat Transfer Enhancement in Dispersed Flows of Large Particles and in Bubbly Flows**

Here we develop a macroscopic model for enhancement of heat transfer due to the presence of a discontinuous phase in a continuous phase. The formulation is limited to flows in which the particles do not penetrate the viscous sublayer. As such the main mode of enhancement of heat transfer is the increased agitation or turbulence created by the particles. For brevity a great deal of calculations and explanations have been omitted.
and we urge interested readers to refer to Ref. 9 for more details.

**Turbulence Intensity in Dispersed Flows**

For dispersed flow in pipes, the turbulence level in the flow is the outcome of the contributions due to wall shear and the presence of particles. The disturbance from the wall, which is referred to as the wall-generated turbulence, is similar to the case of single-phase flow. The disturbance of the particles, on the other hand, is through their inertial interaction with the continuous phase.

Generally a particle that is entrained in a fluid is subjected to the drag force due to the relative velocity between the particles and the fluid, the pressure force due to the pressure gradient applied to the fluid and the hydrostatic head, the Basset force due to the presence of a nonuniform flow around the particles, the force due to interaction between the particles, the virtual mass effect, and the body force. The evaluation of the Basset force, the virtual mass effect, and the particle interaction force requires a detailed knowledge of the flow field around the particle and is beyond the scope of this work. However, since this analysis is intended merely to find a force scale for estimating the drag on a particle, the Basset force, the interaction force, and the force due to the virtual mass effect have been neglected. Thus, it is assumed that:

1. The number density of the particles is sufficiently small that the effect of the presence of other particles on the flow field around a particle can be ignored.
2. The most dominant forces on the particles are the drag force, the weight of the particles and the force due to pressure difference across the particles.
3. Only the hydrostatic head of the fluid contributes to the pressure variation around the particles.

**Figure 5.** Two-phase relative enhancement along rod bundle at Re = 17,000 for 100 µm particles.

The first assumption is violated when the number density of the particles is large. As has been shown in Ishii and Zuber [10], proximity of other particles can influence the drag on a single particle. The limitation arising out of this assumption will be clear when the model predictions are compared with the data. The third assumption is of little consequence as long as the absolute value of the density difference between the continuous and discontinuous phases is large and there is not a large pressure gradient along the direction of the flow.

Under the above assumptions, the drag and buoyancy forces will equal each other after the particle has attained its terminal velocity. If \( V_p \) is the volume of the particle, the drag force on the particle can thus be written as

\[
D_i = \rho_s - \rho_d \frac{V_p}{2} \rho \Delta \rho \frac{D_i}{A_w}
\]

If \( n_p \) is the number of particles contained in a given volume of the flow bounded by the tube wall, the total drag force on the particles can be written as

\[
F_{D_t} = n_p D_i
\]

As an overall scale for the magnitude of the wall-generated turbulence, it is common in turbulence studies to use the friction velocity

\[
(u'^2)_{1/2} = \frac{\tau_w}{\rho}
\]

Now if we assume that \( A_w \) is the circumferential area of the wall, an equivalent shear stress can be defined as

\[
\tau_{id} = n_p D_i / A_w
\]

which resembles the frictional effect of the particles on the flow. By invoking a similarity between the wall-generated turbulence and the turbulence due to the presence of the discontinuous phase, an interfacial friction velocity representing the level of turbulence intensity generated by the dispersed phase can be defined as

\[
(u'^2)_{1/2} = \frac{\tau_{id}}{\rho_c}
\]

Next a basic assumption is made that the turbulence due to the wall and the dispersed phase can be linearly superimposed. Thus for the two-phase flow,

\[
(u'^2)_{1/2} = (u'^2)_{1/2} + C_1(u'^2)_{1/2}
\]

where \( C_1 \) is an empirical parameter that can vary in the radial direction but should have a value on the order of unity. The above formulation is different from that of Theofanous and Sullivan [8], who superimposed the turbulence energy of the wall-generated disturbance and that of the particles.

Dividing both sides of Eq. (15) by \( (u'^2)_{1/2} \), yields

\[
\frac{(u'^2)_{1/2}}{(u'^2)_{1/2}} = 1 + C_1 \frac{(u'^2)_{1/2}}{(u'^2)_{1/2}}
\]

**Figure 6.** Correlation of relative enhancement of heat transfer with \( n_p^{2/3} \), \( \delta_p \rho_d \) for 30 µm and 100 µm particles.
But
\[
\left( \frac{u^*_{10}}{u^*_{10}} \right)^{1/2} = \left( \frac{u^*_{10}}{u^*_{10}} \right)^{1/2} \quad (u^*_{10})^{1/2} \quad (u^*_{10})^{1/2}
\]
which results in
\[
\left( \frac{u^*_{10}}{u^*_{10}} \right)^{1/2} = 1 + C_1 \left( \frac{\tau_{id}}{\tau_w} \right)^{1/2}
\]
(18)
The wall shear stress \( \tau_w \) for the continuous phase is the same as in a single-phase flow with a velocity equal to the superficial velocity of the two-phase dispersed flow,
\[
\tau_w = \frac{C_f}{2} \rho \bar{u}^2
\]
(19)
Then, by using Eqs. (13) and (19) it can be shown that
\[
\frac{\tau_{id}}{\tau_w} = \frac{1}{2C_f} \left( \frac{\beta \rho_c - \rho_d \bar{u}^2 D}{\rho \bar{u}^2} \right) \left( \beta \frac{Gr}{Re^2} \right)
\]
(20)
where \( \beta \) is the volume fraction of the particles. The evaluation of \( \beta \) involves the terminal velocity of the particles [9], which itself depends on the direction of flow. Substitution of Eq. (20) into Eq. (18) results in
\[
\left( \frac{u^*_{10}}{u^*_{10}} \right)^{1/2} \left( \frac{u^*_{10}}{u^*_{10}} \right)^{1/2} = 1 + C_2 \left( \beta \frac{Gr}{Re^2} \right)^{1/2}
\]
(21)
where \( C_2 = C_1/(2C_f)^{1/2} \). Interestingly, the dimensionless group obtained here is the same as that obtained by Drucker et al [7].

In terms of the single-phase friction velocity, Eq. (3), due to Theofanous and Sullivan, reduces to
\[
\left( \frac{u^*_{10}}{u^*_{10}} \right)^{1/2} = \left[ (1 - \beta) (1 - \beta) + \frac{\lambda}{2C_f} \left( \beta - \beta^2 \frac{Gr}{Re^2} \right) \right]^{1/2}
\]
(22)
It is seen that the two equations are similar in the limit of small \( \beta \). At large \( \beta \) both models break down because of strong particle–particle interactions and changes in the flow pattern. Theofanous and Sullivan also measured the turbulence intensity in bubbly flows, and in Fig. 8 predictions from Eq. (21) are compared with their data. In plotting the predictions, the base flow was assumed to be fully turbulent such that \( C_f \) had a value of 0.005. It is noted that the data and predictions compare fairly well when \( C_f \) is assumed to have a value of 1.2. A value of \( C_f \) close to unity suggests that the assumption regarding the superposition of the turbulence intensities is not unrealistic.

Dispersed Flow Heat Transfer Review of the Three-Layer Model of Single-Phase Turbulence

The fully developed energy equation for single-phase turbulent flow in a cylindrical tube can be written as
\[
\frac{\alpha}{\beta} \frac{d\bar{T}}{dz} = \frac{1}{\beta} \frac{\partial}{\partial r} \left[ r(\alpha + \epsilon_H) \frac{\partial T}{\partial r} \right]
\]
(23)
Investigation of Dispersed Flow Heat Transfer

If we assume that the velocity gradient is confined to a very narrow region near the wall, the velocity $u$ can be replaced by $U$, the bulk velocity of the flow. With this assumption, the integration of Eq. (23) combined with the overall energy balance yields

$$T - T_w = -q''/\rho C_p \int_0^y 1 - y/R \frac{1}{\nu/\Pr + \epsilon_H} dy $$

where $y$ is the distance from the tube wall. By defining the dimensionless temperature and distance as

$$T^+ = \frac{T - T_w}{q''/\rho C_p}, \quad y^+ = \frac{yu^*}{v} $$

Eq. (24) reduces to

$$T^+ = \int_0^{y^+} 1 - y^+/R^+ \frac{1}{1/\nu + \epsilon_H/v} dy^+ $$

In evaluating the integral in Eq. (26), the forms of eddy diffusivity in the viscous sublayer, buffer layer, and turbulent core are chosen as

$$\epsilon_H/v = 0, \quad y^+ < \delta_1 $$

$$\epsilon_H/v = K_1 y^+, \quad \delta_1 \leq y^+ < \delta_2 $$

$$\epsilon_H/v = K_2 y^+, \quad y^+ \geq \delta_2 $$

In the above equations, $a_0$ and $a_02$ are constants that are obtained by matching $\epsilon_H/v$ at $y^+ = \delta_1$ and $y^+ = \delta_2$ in Eqs. (27), (28) and (29).

$$a_01 = -K_1 \delta_1 $$

$$b_01 = K_1 (\delta_2 - \delta_1) - K_2 \delta_2 $$

**Modification of Three-Layer Model for Two-Phase Flows** In carrying out the heat transfer analysis for the dispersed flow, some simplifications are possible because the following assumptions were found to be valid:

1. Effect of the thermal capacity ($\rho C_p$) of the discontinuous phase is negligible. In bubbly water flow, where ($\rho C_p$)air $\ll$ ($\rho C_p$)water, this assumption is well justified. In solid-air flow, on the other hand, the dispersed phase is similar to a volumetrically distributed heat sink throughout the fluid. For turbulent flows, which have a relatively flat velocity profile, this effect tends to decrease the temperature of the wall and the fluid equally. Consequently, the overall effect of $\rho C_p$ of particles on the heat transfer coefficient, which is defined as $q''/(T_w - t_0)$, is negligible.

2. The amount of thermal energy that is conducted into the particles during their collision with the heated wall is negligible. In air–water flows, where ($\rho C_p$)air $\ll$ ($\rho C_p$)water, and for solid–air flows, where the contact time with the wall is very short, this is a realistic assumption.

Once these two assumptions are made, it becomes clear that the most significant influence of the presence of the dispersed phase on the heat transport is through the enhancement of turbulence within the continuous phase.

**Turbulence Near the Wall** As long as the diameter of the particles of the dispersed phase is larger than the viscous sublayer thickness, or the number density of the particles is small, it is reasonable to assume that the particles appear near the wall very infrequently. This assumption is in agreement with the observations of Lee and Durst [6] for solid particles in the size range of 100–800 μm.

Accordingly, the energy equation for the turbulent dispersed flow in the near-wall region would be identical to Eq. (23) for single-phase flow.

$$\bar{u}_{wall} \frac{d T_w}{dz} = \frac{1}{r} \frac{\partial}{\partial r} \left[ r (\alpha + \epsilon_{20}) \frac{\partial T}{\partial r} \right] $$

where the subscript “wall” refers to the wall region. For a consistent comparison between the single-phase and two-phase heat transfer coefficients, it is important to consider equal mass flow rates of the continuous phase for both cases so that

$$\bar{u}_{wall} = U $$

In order to develop an expression for $\epsilon_{20}$ in the near-wall region, the following assumptions are made:

1. The eddies created by the particles penetrate the viscous sublayer. This assumption is dependent on the inertia of the continuous phase and on the axial inertia of the particles.

2. The thickness of the viscous sublayer is not influenced by the dispersed phase. Therefore the characteristic friction velocity is the same as for single-phase flow.

3. The presence of a dispersed phase tends to shrink the buffer layer. The thinning of the buffer layer depends on the volumetric fraction of the particles or bubbles. If $\delta_1$ and $\delta_2$ are the thicknesses of the viscous and buffer layers for the single-phase flow, the thickness of the buffer layer in the presence of the particles is written as

$$\delta_b = \delta_1 + (1 - \beta) \delta_2 $$

According to this expression, in the limit $\beta \to 0$, the buffer layer thickness is the same as for single-phase flow.

4. The characteristic friction velocity for the buffer layer and for the turbulent core is that for the two-phase flow. However, when the inertia of the continuous phase be-
comes high or the particles are confined to the turbulent core, the friction velocity in the buffer layer is that for a single-phase flow.

The intensity of the eddies penetrating the viscous sublayer will decrease towards the wall. Thus the level of disturbance will be zero at the wall and will approach a value consistent with the buffer layer at the edge of the viscous layer. As a first approximation, the eddy diffusivity in the viscous sublayer can be assumed to vary linearly with distance as

$$\frac{\epsilon_{wall}}{\nu} = K^* y^+, \quad y^+ < \delta_1$$

(35)

where $y^+ = y u^*/\nu$.

To determine $K^*$, we utilize the observation made in single-phase flows that there is a close conformity between turbulent kinetic energy and turbulent shear stress. If we assume that the same is true in the wall region for the dispersed flow, the shear stress and kinetic energy representative of eddies originating from the discontinuous phase can be written in terms of those for single-phase flow as

$$\frac{(u''v'')_{wall}}{(u''v'')_{10}} = \frac{\epsilon_{wall}(d\alpha/dy)_{wall}}{\epsilon_{10}(d\alpha/dy)_{10}}$$

(36)

In the above equation, the ratio $u''v''/u'_{10}^2$ is a measure of the relative increase in turbulence energy of the buffer layer and consequently of the level of disturbances that penetrate into the viscous sublayer. The ratio of shear stresses can also be written as

$$\frac{(u''v'')_{wall}}{(u''v'')_{10}} = \frac{\epsilon_{wall}(d\alpha/dy)_{wall}}{\epsilon_{10}(d\alpha/dy)_{10}}$$

(37)

In Eq. (37), $(d\alpha/dy)_{wall}/(d\alpha/dy)_{10}$ is of the order of unity. As such, Eq. (36) and (37) yield

$$\frac{\epsilon_{wall}}{\epsilon_{10}} = \frac{(u''v'')_{wall}}{(u''v'')_{10}} = \frac{u''v''}{u'_{10}^2}$$

(38)

But it was shown earlier that

$$\frac{u''v''}{u'_{10}^2} = \frac{\beta}{Re^2}$$

(39)

Thus,

$$\frac{\epsilon_{wall}}{\epsilon_{10}} = \frac{b\beta}{Re^2}$$

(40)

where $b$ is the constant of proportionality. In single-phase flow, $\epsilon_{10}$ in the vicinity of the wall can be written as

$$\epsilon_{10} = \nu K_1 y^+$$

(41)

where $K_1$ is a constant having a value of 0.2. Combining Eqs. (40) and (41), an expression for eddy diffusivity in the viscous sublayer is obtained:

$$\frac{\epsilon_{wall}}{\nu} = b \left( \frac{\beta}{Re^2} \right) K_1 y^+$$

(42)

Comparison of Eq. (42) with Eq. (35) shows that

$$K^* = b \left( \frac{\beta}{Re^2} \right) K_1$$

(43)

In the buffer layer, the eddy diffusivity varies with distance in the same way as for single-phase flow except that now the friction velocity is that for the two-phase flow. Thus in the buffer layer

$$\frac{\epsilon_{wall}}{\nu} = K^* y^+, \quad y^+ < \delta_1$$

(44)

where $y^+ = y u^*/\nu$. Subsequently by using an integration procedure similar to the one that led to Eq. (26) for single-phase flow, the temperature in the near-wall region of the dispersed flow is obtained as

$$T^+ = \frac{1}{Pr} \frac{1}{1/R^+} \int_0^{\delta_b} dy^+ \left( 1 - y^+ / R^+ \right)$$

(45)

where $\epsilon_{wall}/\nu$ is given by Eq. (42) and (44).

**Turbulence Away From the Wall** In the turbulent core of a dispersed flow, the particles move in all directions. Quantitative analysis of the effects of particles on the heat transport can be simplified, if, according to Fig. 9, it is assumed that all particles pass through a group of channels along their axial motion. Within the channels, it is believed that intense agitation of the fluid by the particles can provide a region of uniform temperature. In other words, the thermal resistance of the channels would be effectively zero.

On the other hand, in the fraction of the fluid where the particles do not physically enter, the disturbance due to the traveling eddies tends to enhance the transport of heat.

Remembering that the thermal capacity of the discontinuous phase can be neglected, the governing energy equation for the region away from the wall can be written as

$$\frac{dT_{core}}{dz} = \frac{1}{r} \frac{\partial}{\partial r} \left[ r(\alpha + \epsilon_{2b}) \frac{\partial T}{\partial r} \right]$$

(46)

In Eq. (46), $J_{core}$ is the superficial velocity of the continuous phase and it is assumed that

$$J_{core} = U$$

(47)

The expression for eddy diffusivity in the core region is written as

$$\frac{\epsilon_{wall}}{\nu} = K_2 y^+ + b_{12}, \quad y^+ \geq \delta_b$$

(48)

with $y^+ = y u^*/\nu$. Integration of Eq. (46) proceeds in a manner...
similar to the single-phase flow case:

\[ T^* = \int_{\beta_0}^{1} \frac{1 - \gamma^*/R^*}{1/\Pr + \epsilon_{\gamma}/\nu} \, d\gamma^* \]  

(49)

However, after taking into account the fraction of the continuous phase that is occupied by channels with zero thermal resistance and a volume of \( \beta \), the expression for \( T^* \) for the core becomes

\[ T^*_{\text{core}} = (1 - \beta) \int_{\beta_0}^{1} \frac{1 - \gamma^*/R^*}{1/\Pr + \epsilon_{\gamma}/\nu} \, d\gamma^* \]  

(50)

It should be noted that Eq. (50) introduces \( \beta \) as an independent parameter. Because of the artificial discontinuity of \( \epsilon_{\gamma}^* \) across the three layers, the matching of \( \epsilon_{\gamma}^* \) at the boundaries was carried out at actual physical distances (not the dimensionless sizes \( \delta_1 \) and \( \delta_b \)). This way, the expressions for \( \epsilon_{\gamma}^* \) would be continuous at the boundaries of the viscous sublayer and buffer layers. It was found that

\[ a_{\epsilon_{\gamma}} = (K^* - K_1) \delta_1 \]  

(51)

\[ b_{\epsilon_{\gamma}} = K^* \delta_1 + \left[ K_1 (\delta_b - \delta_1) - K_2 \delta_b \right] X \]  

(52)

where

\[ X = \frac{u_{\epsilon_{\gamma}}^*}{u_{\epsilon_{\gamma}}^*} \]  

(53)

Equations (45) and (50) can now be integrated after appropriate expressions for \( \epsilon_{\gamma}^* \) from Eqs. (42), (44), and (48) are substituted in them. As a simple and convenient way to calculate the heat transfer coefficient, one can use

\[ h = \frac{q^*}{T_w - T_c} \left( \frac{T_w - T_c}{T_w - T_b} \right) \]  

(54)

where \( T_c \) is the temperature at the centerline of the pipe. Subsequently the ratio of the two-phase to single-phase heat transfer coefficients would be obtained as

\[ \psi = \frac{h_{2\beta}^*}{h_{1\beta}^*} = (T_w - T_c)_{1\beta} / ((T_w - T_c)/(T_w - T_b))_{1\beta} \]  

(55)

The ratio

\[ [(T_w - T_c)/(T_w - T_b)]_{1\beta} / [(T_w - T_c)/(T_w - T_b)]_{2\beta} \]

is generally very close to unity. By incorporating Eqs. (26), (45), and (50) into Eq. (55), it is found that

\[ \psi = \frac{\Gamma_1 + \Gamma_2 + \Gamma_3}{\Lambda_1 (u_{\epsilon_{\gamma}}^* / u_{\epsilon_{\gamma}}^*) + \Lambda_2 (1 - \beta) \Lambda_3} \]  

(56)

where

\[ \Gamma_1 = \frac{1}{K_1} \left[ \left( 1 + \frac{A_{16}}{R^*} \right) \left( 1 + \frac{B_{16}}{R^*} \right) \right] \]  

(57)

\[ \Gamma_2 = \frac{1}{K_2} \left[ \left( 1 + \frac{A_{26}}{R^*} \right) \left( 1 + \frac{B_{26}}{R^*} \right) \right] \]  

(58)

\[ \Gamma_3 = \frac{1}{K_{16}} \left[ \left( 1 + \frac{A_{16}}{R^*} \right) \left( 1 + \frac{B_{16}}{R^*} \right) \right] \]  

(59)

\[ \Lambda_1 = \frac{1}{K_{16}} \ln \left( \frac{1/\Pr + \epsilon_{\gamma}^*}{1/\Pr} \right) \]  

(60)

\[ \Lambda_2 = \frac{1}{K_2} \left[ \left( 1 + \frac{A_{26}}{R^*} \right) \left( 1 + \frac{B_{26}}{R^*} \right) \right] \]  

(61)

\[ \Lambda_3 = \frac{1}{K_2} \left[ \left( 1 + \frac{A_{26}}{R^*} \right) \left( 1 + \frac{B_{26}}{R^*} \right) \right] \]  

(62)

\[ A_{16} = \frac{1}{K_1} + a_{16} \]  

(63)

\[ B_{16} = \frac{b_{16}}{K_1} \]  

(64)

and

\[ R^* = \frac{Re u_{\epsilon_{\gamma}}^*}{2U} \]  

(65)

The parameters related to the structure of turbulence are assumed to have the following values:

\[ \delta_1 = 5, \quad \delta_2 = 30, \quad K_1 = 0.2, \quad K_2 = 0.4 \]

\[ \frac{u^*}{U} = \left( \frac{C_f}{2} \right)^{1/2} = 0.05, \quad b = 0.6 \]

The values chosen for the parameters are the same as those commonly used for single-phase flows. The parameter \( b \) is unique to two-phase flow, and its value is obtained empirically.

In Fig. 10, the predictions for \( \psi \) are compared with the air-water flow data of Verschoor and Stemerding [11]. The data

![Figure 10](image-url)
are for Pr = 6.7 and are plotted against $\beta \frac{Gr}{Re^2}$. Also, in the figure the predictions from Eq. (5) (Drucker et al. [7]) are presented for comparison. It should be stressed here that in the present model $\beta$ and $\beta \frac{Gr}{Re^2}$ are two independent parameters, whereas in the work of Drucker et al $\beta \frac{Gr}{Re^2}$ was the only independent parameter. It is seen from Fig. 10 that there is a good agreement between the predictions from the present model and the data.

Referring to Fig. 11, a similar comparison is made with the bubbly flow data of Michiyoshi [12] for Pr = 6.56 and 16,500 < Re < 50,000 where the predictions of the model consistently agree with the experimental observations.

In Figs. 12 and 13, the air–water flow data of Kudirka et al [13] and of Groothuis and Hendal [14], respectively, are compared with the present model. Kudirka et al’s data are for Pr = 6.2 and 24,000 < Re < 48,000, whereas the data of Groothuis and Hendal are for Pr = 3.03 and 8300 < Re < 11,600. It is seen that although there are few data for Pr = 3.03, in both cases predictions from the model consistently agree with the experimental observations.

**Modifications of the Model for High Reynolds Numbers and High Density Ratio of Discontinuous to Continuous Phase**

When the inertia of the continuous phase becomes very large or the particles have a density sufficiently higher than that of the continuous phase that the axial inertia of the particles is much greater than their transverse inertia, the particle-generated eddies will be confined to the core region. Thus, it is plausible to assume that although the particle-generated eddies will shrink the buffer layer, they will not penetrate the buffer or the viscous layers. As a result, the structure of the turbulence in the vicinity of the wall will be the same as for single-phase flow. For flow Reynolds numbers greater than 100,000 and $\rho_d/\rho_c \gg 1$, the functional terms of eddy diffusivity in the three regions are taken to be

\[ \frac{\varepsilon_{y}}{\nu} = 0, \quad y^+ < \delta_1 \]

\[ \frac{\varepsilon_{y}}{\nu} = K_1 y^+ + a_{02}, \quad \delta_1 \leq y^+ < \delta_h \]  

with

\[ y^+ = y u^+_{1o}/\nu \]  

and for the turbulent core,

\[ \frac{\varepsilon_{y}}{\nu} = K_2 y^+ + b_{02}, \quad y^+ \approx \delta_b \]  

with

\[ y^+ = y u^+_{2o}/\nu \]

Also, the continuity of eddy diffusivity at the boundaries of the layers requires that

\[ a_{02} = -K_1 \delta_1 \]

\[ b_{02} = K_1 (\delta_b - \delta_1) - K_2 \delta_b X \]

Use of the above expressions for eddy diffusivity in Eqs. (26), (45), and (50) for the temperature distribution finally leads to an expression for the ratio of two-phase to single-phase heat transfer coefficients as

\[ \Theta = \frac{h_2}{h_1} = \frac{\Gamma_1 + \Gamma_2 + \Gamma_3}{\Gamma_1 + \Gamma_2} \left( \frac{u^+_{1o}}{u^+_{iso}} \right) \]  

\[ \frac{Pr = 6.2}{24,000 < Re < 48,000} \]  

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**Figure 11.** Enhancement of heat transfer coefficient for air–water flow with Pr = 6.56.

**Figure 12.** Enhancement of heat transfer coefficient for air–water flow with Pr = 6.2.
3.0
Pr = 3.03
8,300 < Re < 11,600
- Groothuis et al. [14]
- Drucker et al. [7]
- Present Model

where

\[ \Lambda_1 = \text{Pr} \delta_1 \]  
\[ \Lambda_2 = \frac{1}{K_1} \left[ \frac{1 + A_{26}}{R^+} \ln \left( \frac{A_{26} + \delta_2}{A_{26} + \delta_1} \right) - \frac{\delta_2 - \delta_1}{R^+} \right] \]  
\[ \Lambda_3 = \frac{1}{K_2} \left[ \frac{1 + B_{26}}{R^+ X} \ln \left( \frac{B_{26} + R^+ X}{B_{26} + \delta_2 X} \right) - \frac{R^+ - \delta_2}{R^+} \right] \]  

In Eq. (73), the expressions for parameters \( \Gamma_1, \Gamma_2, \) and \( \Gamma_3 \) are the same as those given in Eqs. (57)-(59).

Figure 14 shows a comparison of the predictions from Eq. (73) with the bubbly flow data of Fedotkin and Zarudnev [15]. The data are for \( \text{Pr} = 1.77 \) and \( \text{Re} \) varying from \( 10^3 \) to \( 1.5 \times 10^5 \). The predictions are seen to compare reasonably well with the modified model.

Predictions of \( \psi \) from the modified model are also compared in Fig. 15 with the two-phase heat transfer data when \( 100 \mu m \) diameter glass particles were entrained in air (\( \text{Pr} = 0.71 \)).

Again the agreement between the data and the model is quite good.

**PRACTICAL SIGNIFICANCE/USEFULNESS**
Although the present experiments were conducted by entraining solids in a gas, the results should be applicable to situations in which liquid droplets are entrained in a lighter phase and the liquid droplets do not wet the heat transfer surface. The model developed in this work identifies a common underlying mechanism for heat transfer enhancement in entrained and bubbly flows. The model also ties together the independent observations of turbulence intensity and heat transfer augmentation in bubbly flows. It should be emphasized that the model is applicable only to dispersed flow of large particles (\( D_p \geq 100 \mu m \)).

**CONCLUSIONS**
1. The presence of a discontinuous phase in a continuous phase enhances the heat transfer coefficient, in the continu-
Figure 15. Enhancement of heat transfer coefficient for glass–air flow with Pr = 0.71.

ous phase. The enhancement in large particles is found to depend on two independent parameters, $\beta$ and $\beta \, \text{Gr/Re}^2$.

2. The enhancement with smaller particles ($D_p = 30 \, \mu m$) is much higher than that with larger particles ($D_p = 100 \, \mu m$).

3. Different mechanisms are responsible for enhancement in large particles ($D_p \geq 100 \, \mu m$) and in small particles ($D_p < 100 \, \mu m$). Small particles penetrate the sublayer. However, for large particles it is the particle-generated eddies that penetrate the viscous sublayer.

4. Predictions from the eddy diffusivity model compare well with the bubbly flow data reported in the literature as well as the present data obtained with glass particles.

5. The extent to which the particle-generated eddies influence the thermal diffusivity in the wall region depends on the inertia of the continuous phase and the ratio of the density of the discontinuous phase to the density of the continuous phase.

NOMENCLATURE

- $A_{rod}$: area of the outer wall of each rod, $m^2$
- $A_w$: wall area of a tube, $m^2$
- $A_{1b}$: parameter defined in Eq. (63), dimensionless
- $A_{2b}$: parameter defined in Eq. (64), dimensionless
- $a_0$, $a_2$: constants, dimensionless
- $B_{1b}$: parameter defined in Eq. (63), dimensionless
- $B_{2b}$: parameter defined in Eq. (64), dimensionless
- $b$: constant of proportionality used in Eq. (40), dimensionless
- $b_{01}$, $b_{02}$: constants, dimensionless
- $C_f$: friction coefficient for single-phase flow, dimensionless
- $C_p$: specific heat, $J/(kg \, K)$
- $C_{1b}$, $C_2$: constants used in Eqs. (15) and (21), dimensionless
- $D_h$: hydraulic diameter, $m$
- $D_f$: drag force on each particle, $N$
- $D_{p_1}, D_{p_2}$: diameter of a solid particle, $m$
- $F_{id}$: resistance force of all particles on the continuous phase, $N$
- $Gr$: Grashof number ($= \rho_c - \rho_d |gD_h^4/\rho_d \, \nu^2|$), dimensionless
- $g$: gravitational acceleration, $m/s^2$
- $h$: convective heat transfer coefficient, $W/(m^2 \, K)$
- $h_{1b}$: fully developed heat transfer coefficient for single-phase turbulent flow, $W/(m^2 \, K)$
- $h_{2b}$: fully developed heat transfer coefficient for two-phase dispersed flow, $W/(m^2 \, K)$
- $I$: electric current, A
- $J_{cool}$: superficial velocity of dispersed flow in the turbulent core, $m/s$
- $K, K_1, K_2$: turbulence parameters, dimensionless
- $K^s$: modified turbulence parameter, dimensionless
- $k_c$: thermal conductivity of the continuous phase, $W/(m \, K)$
- $k_f$: thermal conductivity of the fluid, $W/(m \, K)$
- $L$: length of the rod, $m$
- $Nu$: Nusselt number ($= hD_h/k_c$), dimensionless
- $Nu_s$: Nusselt number ($= hD_h/k_c$), dimensionless
- $n_p$: number of particles per unit volume, dimensionless
- $Pr$: Prandtl number ($= \nu/\alpha$), dimensionless
- $pd$: penetration depth of the particles as used in Eq. (2), $m$
- $q^2$: kinetic energy of turbulence
- $q^s$: local wall heat flux, $W/m^2$
- $R$: Radius of the tube or rod, $m$
- $Re$: Reynolds number, dimensionless
- $R_o$: ohmic resistance of the rods, ohms
\[ R^+ \] dimensionless radius
\[ r \] radial distance, m
\[ T \] temperature, K
\[ T_b \] bulk temperature, K
\[ T_c \] centerline temperature, K
\[ T_w \] wall temperature, K
\[ T^+ \] dimensionless temperature
\[ t \] time, s
\[ U \] bulk velocity of turbulent single-phase flow, m/s
\[ \bar{U}_r \] radial average velocity of two-phase flow, m/s
\[ \bar{u} \] timewise average velocity of turbulent flow, m/s
\[ \bar{u}_{wall} \] average velocity of the flow in the wall region, m/s
\[ u' \] fluctuating velocity of turbulent flow, m/s
\[ u'_{id} \] fluctuating velocity of the disturbance due to presence of particles, m/s
\[ u'_{bo} \] fluctuating velocity of turbulence in single-phase flow, m/s
\[ u'_{so} \] fluctuating velocity of turbulence in two-phase dispersed flow, m/s
\[ \bar{u} \bar{v}' \] turbulent shear stress
\[ u^* \] friction velocity in single-phase turbulent flow, m/s
\[ u^*_b \] interfacial friction velocity due to disturbance created by particles, m/s
\[ u^*_o \] friction velocity in single-phase turbulent flow, m/s
\[ u^*_s \] friction velocity in two-phase dispersed flow, m/s
\[ V_p \] volume of a particle, m³
\[ W_a \] mass flow rate of air, kg/s
\[ W_s \] mass flow rate of solid particles, kg/s
\[ y \] distance from the wall, m
\[ y_1 \] outer distance of viscous sublayer from the wall, m
\[ y_2 \] outer distance of buffer layer from the wall in two-phase dispersed flow, m
\[ y^+ \] dimensionless distance from the wall
\[ z \] axial distance along the wall

Greek Symbols
\[ \alpha \] thermal diffusivity \( (k_f/\rho C_p) \), m²/s
\[ \beta \] volumetric fraction of the discontinuous phase, dimensionless
\[ \Gamma_1, \Gamma_2, \Gamma_3 \] parameters defined in Eqs. (57)–(59), dimensionless
\[ \delta_b \] dimensionless outer distance of the buffer layer in two-phase dispersed flow
\[ \delta_o \] dimensionless thickness of the viscous sublayer in Briller’s model
\[ \delta_t \] dimensionless thickness of the viscous sublayer in single-phase and two-phase turbulent flow
\[ \delta_d \] dimensionless outer distance of the buffer layer from the wall in single-phase turbulent flow
\[ \varepsilon_H \] turbulent thermal diffusivity, m²/s
\[ \varepsilon_{wall} \] turbulent thermal diffusivity in the wall region, m²/s
\[ \varepsilon_{bo} \] turbulent thermal diffusivity in single-phase flow, m²/s
\[ \varepsilon_{so} \] turbulent thermal diffusivity in two-phase dispersed flow, m²/s
\[ \Lambda_1, \Lambda_2, \Lambda_3 \] parameters defined in Eqs. (60)–(62) and (74)–(76), dimensionless
\[ \lambda \] constant
\[ \nu \] kinematic viscosity, m²/s
\[ \rho \] density, kg/m³
\[ \tau_{id} \] interfacial shear stress, N/m²
\[ \tau_w \] shear stress on the wall, N/m²
\[ X \] defined as \( u^*_o/u^*_p \)
\[ \psi \] ratio of fully developed two-phase heat transfer coefficient to fully developed single-phase heat transfer coefficient
\[ \psi_c \] ratio of local two-phase heat transfer coefficient to fully developed single-phase heat transfer coefficient

Subscripts
\[ a \] air
\[ b \] bulk
\[ c \] continuous phase
\[ core \] core region
\[ d \] discontinuous phase
\[ f \] fluid
\[ H \] heat
\[ p \] particle
\[ s \] solid particle
\[ wall \] wall region
\[ z \] axial distance along the wall
\[ \phi \] single-phase
\[ \phi_2 \] two-phase

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